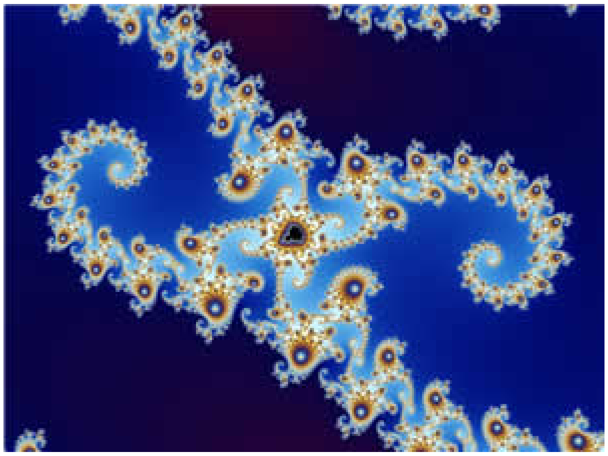
5.1: Simple Arithmetic

Complex Numbers



Complex numbers are not called "complex" because they are difficult. It means that they are made up of two different types of numbers, just like a "building complex" refers to a bunch of buildings joined together.

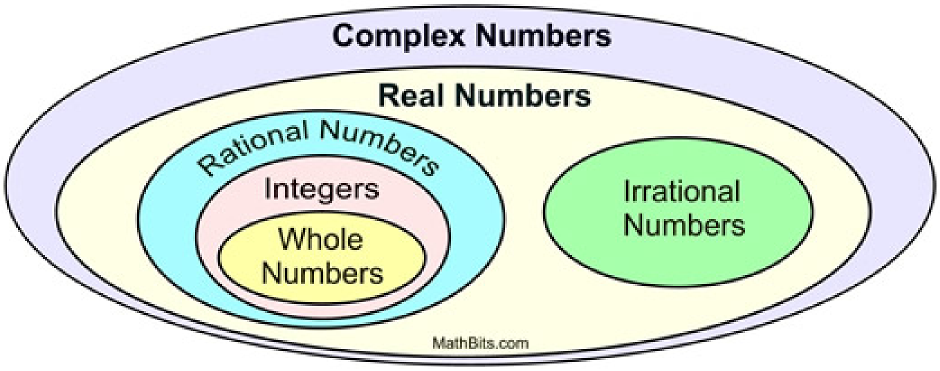
A **Complex Number** is a combination of a Real Number and an Imaginary Number

The standard form of complex numbers is $a + bi$ where a stands for a real number and b stands for the imaginary part. Remember that $i = \sqrt{-1}$ ... and that $i^2 = -1$!

- Examples: $-2 + 4i$ $\quad 3 - 5i$ $\quad 0.6 + 1.3i$ $\quad -2 + \pi i$ $\quad \sqrt{3} + \sqrt{7}i$

If you really think about it, ALL numbers are complex numbers. This is hard for some students to understand, but you may just have to put a zero in for "a" or "b" to see it more clearly.

- More examples: $4i$ $\quad -5i$ $\quad 1.3i$ $\quad -2$ $\quad \sqrt{7}$



II. Complex Arithmetic

Adding, subtracting and multiplying complex numbers is just like arithmetic with Real numbers: You combine like terms – real numbers with real numbers and imaginary numbers with imaginary numbers.

EXAMPLE 1: Add two complex numbers

(7 + i)+(3 -2 i)	
7 + 3 + i - 2i	Remove parentheses
10 - 2i	Combine real and imaginary parts
10 – 2i	ANSWER

Your Turn:

1. Add or subtract the following. Write the answer in standard form.
   a. (6 – i) + (7 + 3i)

   b. (12 + 4i) – (3 – 7i)

   c. (12 – 3i) + (7 + 3i)

   d. 7 – (3 + 4i) + 6i

   e. -10 - (6 – 5i) -9i

To multiply two complex numbers, simply use the distributive property, just as you do with real numbers and algebraic expressions.

Don't forget that since $= \sqrt{-1}$, $i^2 = -1$, and you should replace i^2 with -1 anytime you see it.

EXAMPLE 2: Multiply two complex numbers

4i (-6 + i)	
$-24i + 4i^2$	Distributive Property
-24i +4(-1)	Use $i^2 = -1$
-4 -24i	Write ANSWER in standard form

2. Multiply the following. Write the answer in standard form (a + bi).
    a. 3i (-5 + 2i)

    b. (3 + 4i) (4 + 2i)

    c. (3 – 2i)(4 + 3i)

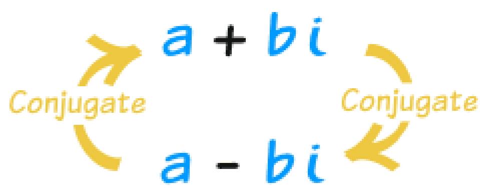
    d. $(3 - 6i)^2$

    e. (3 + 4i) – 2(7 – 5i) + 2i(9 + 12i)

    f. $(2 + 4i^5) + (1 - 9i^6)$

III. Dividing Complex Numbers

The **conjugate** of a complex number is when you switch the sign of the complex part, as see below:



The notation for a conjugate is a bar over the term(s):

- If w = 7 + 3i, then $\overline{w} = 7 - 3i$.
- If z = 9 – 2i, then $\bar{z} = 9 + 2i$.

3. Find the complex conjugates of the following:
    a. 6 + 2i
    b. -4 + i
    c. 3 - 7i
    d. 5i + 3 (careful!)

4. Multiply the following complex conjugates.
    a. (3 + i)(3 - i)

    b. (4 - 2i)(4 + 2i)

    c. (3 + 5i)(3 - 5i)

    d. (a + bi)(a - bi)

5. Write a conjecture that describes the product of two complex conjugates.

To divide complex numbers, you will need to identify conjugates and then multiply the numerator and denominator by the conjugate of the denominator and simplify. Let's look at some examples:

EXAMPLE 1: Divide $\frac{3-8\mathrm{i}}{5+\mathrm{i}}$.

$\frac{3-8i}{5+i} \cdot \frac{5-i}{5-i}$	multiply numerator and denominator by conjugate of denominator
$\frac{15-5\mathrm{i}-40i+8i^2}{25-i^2}$	Distributive Property (or shortcut rule)
$\frac{7-45\mathrm{i}}{26}$	simply numerator and denominator (combine like terms, $\mathrm{i}^2 = -1$
$\frac{7}{26} - \frac{45}{26}i$	write your answer in standard form
$\frac{7}{26} - \frac{45}{26}i$	simplify (not possible here)

<u>Your Turn</u>:

6. Divide the following. Write the answer in standard form.
    a. $\frac{5+i}{2-4i}$

    b. $\frac{11-5i}{2-4i}$

    c. $\frac{8+2i}{5+i}$

IV. Application

7. In an electrical circuit, the voltage, V, is given by the formula $V = IZ$, where I is the current and Z is the impedance. Both the current and impedance are represented by complex numbers. Find the impedance if the current is 3 + 2i and the voltage is 14 + 5i.

Summary

If you have mastered the concept of dividing complex numbers, you should feel confident in your knowledge of the following main points of this lesson. Let's review:

- Complex numbers in the form a + bi are considered to be in standard form where "a" is the real part and "b" is the imaginary part.
- ALL numbers can be considered complex numbers.
- To find the conjugate of a complex number, you simply change the sign of the imaginary part.
- A short cut or rule for multiplying complex numbers is to square the real part and the imaginary part and then add them together [$(a + bi)(a - bi) = a^2 + b^2$]. Their answer when multiplied will always be a real, positive, whole number.
- To divide complex numbers, you must multiply the numerator and denominator by the conjugate of the denominator.
- Answers when dividing complex numbers should always be given in standard form and reduced when possible.